Dark Matter, CP Problem and Axion Solution

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This article investigates how cosmology, particle physics, and electromagnetism intersect at an essential proposition to dark matter: a particle called the ”Axion”. An axion is described by significant mathematical viewpoints using Quantum Field Theory. Axion is akin to an overture to a new paradigm in physics; it is a solution to a symmetry problem in the physics of the strong interaction that occurs inside the atomic nucleus. It will be described using certain symmetries and fundamental concepts in electromagnetism, such as dipoles. Furthermore, axion’s early universe appearance scenarios will also be addressed.

Introduction

The Axion idea emerged nearly in the 1980s because of the thought that it could be a candidate for dark matter. It is related to Quantum Chromodynamics (QCD), which is a type of quantum field theory that explains the strong interactions between quarks and gluons. Charge Parity (CP) symmetry is the conserved symmetry of the system when charge conjugation (C) and parity transformations (P) are applied, which will be further explained in the ”Symmetries” subsection. In QCD, it is observed that this symmetry is never violated and the proposition to why it is never violated is the axion. This proposition is also a candidate for dark matter. Dark matter is a form of matter that is not able to generate light, but it can be detected gravitationally. One of the propositions is that it may ”populate the universe in a wave-like state,” which is cold and moving very slowly. Axion complies with this wave-like property since its field is described as oscillatory. Hence, it is one of the major reasons it is a candidate for dark matter [1].

What is Dark Matter

Dark matter is one of the most prominent topics in both cosmology and particle physics. It is known that the total mass of dark matter and visible (normal) matter comprises 32% of the universe, 26% dark matter, and 6% visible matter. The remaining 68% is composed of dark energy [1]. Dark matter is different from visible matter. Visible matter shapes stars and generates light that can be detected through telescopes. Dark matter is a matter that has mass, but it neither directly forms big clusters and formations like visible matter nor interacts with electromagnetic waves.

This is because its interaction with visible matter is only through gravity. It has a major role in the formation of galaxies. Dark matter haloes (spherical regions) are formed by the gravitational collapse of small cosmological structures. This occurs hierarchically, meaning that this repeats over time to form bigger structures. Consequently, galaxies form at the center of dark matter haloes. In the following figure (Fig. 1), this phenomenon is shown by color density. The darker (green) regions of the color show that there are galaxy formations, whereas the lighter (yellow) regions correspond to the early stage: dark matter haloes before galaxy formations [2].

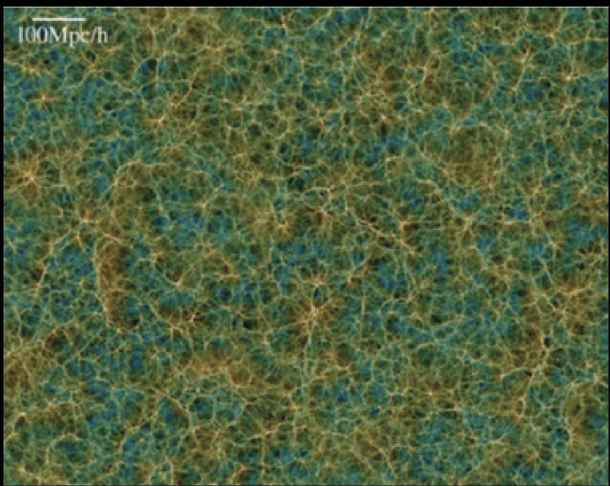


Fig 1. Distribution of dark matter over cosmological scales in the field of galaxy formation [3]

Expansion of the Universe

Dark matter and dark energy are dissimilar concepts, even though they may look similar. Dark energy is the primary reason why the universe continues to expand. Einstein demonstrated that the rate of expansion of the universe is determined by the energy density and total matter, and he postulated that as the universe expands, the total matter density (comprising both dark and visible matter) decreases. The reason is that the volume increases, but the total matter density is the same everywhere. However, the dark energy density should remain constant because, as the volume increases, dark energy will still be present in the same proportion in the new volume, as it plays the role of a cosmological constant. Therefore, the dark energy fraction increases when compared to the total matter density, and the universe will expand, and the rate of expansion will increase. Dark matter slows down the expansion rate by its mass and gravity. So it does not cause a positive acceleration in the expansion of the universe [4].

Expectations and Observations

There have been some experiments conducted on the existence of dark matter. One of these experiments was in 1959. Louise Volders had measured the velocity of hydrogen gas in the galaxy "Triangulum", using hydrogen's atomic transitions and the "redshift" phenomenon, which is described as a decrease in frequency. According to Newton's second law, orbit velocity is proportional to the square root of the total mass; the total amount of matter inside the orbit of the hydrogen gas. He found that the velocity of the hydrogen gas was higher than expected, because the total mass of the stars inside the orbit in Triangulum is not able to provide that high speed value. Therefore, it was concluded that there should be more mass than the total mass of the stars, which is dark matter. In Fig. 2, when a graph of velocity versus distance from the center of the galaxy is plotted, the dotted line describes the expected graph and the solid line describes the measured values plotted on the graph [4]. In the following subsections, a candidate for dark matter," axion," will be presented. Before, some fundamental concepts will be introduced.

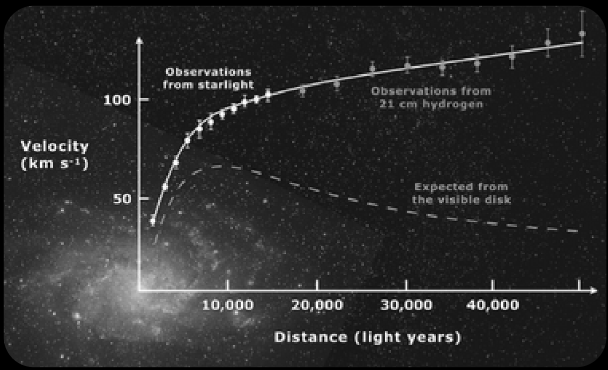


Fig 2. Rotation curve of Triangulum [4]

FUNDAMENTAL CONCEPTS

Transformations

In nature, there are some transformations that do not violate the laws of physics. These transformations are: time transformation, parity transformation, and charge conjugation. Time transformation involves reversing the flow of time (i.e., a video of a pendulum swinging back). Parity transformation reverses the spatial coordinates. It is similar to taking mirror symmetry. For instance, the (x,y,z) coordinate becomes (-x,-y,-z). Charge conjugation is the transformation of a particle into its antiparticle by reversing all of its charges [5]. There are many examples to this phenomenon: The electrical charge +e ($1.602176634 \times 10^{-19}$ C) [6] becomes -e, the proton becomes an antiproton, the electron e− becomes positron e+, and so on.

Symmetries

There are some symmetries that emerge due to these physical transformations. One of them is the charge parity (CP) symmetry. In this symmetry, every particle is replaced with its antiparticle, and a mirror reflection is applied to its spatial coordinates. Another symmetry is the charge parity-time reversal symmetry (CPT). This symmetry includes the time reversal transformation in addition to the properties of CP. Also, it has never been observed to be violated in experiments [1].

Lagrangian

Lagrangian is described as the fact that a physical path will be chosen in order to minimize the time integral of the difference between the kinetic and potential energies.
It is given by the equation:

$$L = T - U \quad (1)$$

where **L** is the Lagrangian, T is the kinetic energy, and U is the potential energy. The intuitive approach to taking this integral is that the kinetic energy will determine the speed of the movement of the system, whereas the potential energy will determine the accumulated energy in the system. Taking their difference will connect these two energies to each other in an equation, which is the Lagrangian. Taking the time integral means finding the general behavior of a system for a given time interval. Having $T - U$ minimized is the meaning of the Lagrangian, because nature chooses the lowest energy state [7].

DIPOLES

What are Dipoles?

A dipole is an underlying concept of electromagnetism in physics. It can be described as the separation of positive and negative charges in atoms or molecules. This happens when there is an electric field inside the specified material, where the electrons and protons are shifted in opposite directions because of the repulsive force they experience [8]. It can also happen inside a current loop where a magnetic field is produced.

Therefore, there are 2 types of dipoles: electric dipole and magnetic dipole, respectively, according to their definitions, because they have a spatial orientation (i.e., position) in space, especially for moving electric dipoles that have trajectories [9], the term ”moment” is used as: electric dipole moment and magnetic dipole moment.

TRANSFORMATIONS ON A NEUTRON

Neutrons

Neutrons are electrically neutral subparticles in an atom's nucleus. Therefore, its separation of positive and negative charges has been a research area in nuclear and particle physics. The electric dipole moment (EDM) of neutrons has not been observed in nature yet. The reason is that it would violate a fundamental symmetry in nature: charge parity symmetry (CP), its reason will be discussed in the following subsections [1].

Transformations

For the time being, in order to prove that the EDM for neutrons is not possible, it will be assumed that neutrons have EDM. In the following observations, if there is a violation, this assumption will be proven wrong, and it can be said that a neutron with EDM is not possible. An electric field and a magnetic field are applied to a neutron having an EDM (i.e., assumption). When the time transformation and parity transformation are also applied to this neutron separately, there are some observations regarding the electric and magnetic field, spin, EDM, and the energy of the neutron. When the time transformation is applied (i.e. the flow of time is reversed) the following phenomenons happen: the electric field stays the same; the magnetic field, spin, EDM, and energy (sign) change. When parity transformation is applied (i.e., spatial coordinates are reversed), magnetic field, spin, and EDM stay the same; electric field and energy (sign) change. Furthermore, it is known that the spin of the neutron and its EDM should align with each other. In Fig. 3, these transformations can be observed [1].

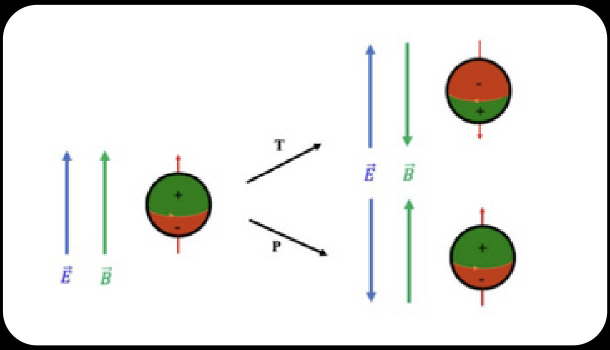


Fig 3. Transformations on a Neutron with electric or magnetic dipole moment: T is time transformation, P is parity transformation, red arrow represents the spin that is proportional to the EDM [1]

Hamiltonian

Hamiltonian is the total energy of a system described by the equation:

$$H = T + U \quad (2)$$

where T is the kinetic energy, and U is the potential energy. It should remain constant in a closed system because of the conservation of energy. The effect of time transformation on the energy of a neutron, which is given by its Hamiltonian, is described by:

$$H = -d \cdot E \quad (3)$$

where H is the Hamiltonian, d is the electrical dipole, and E is the electrical field [6].

CP PROBLEM

CP Problem

According to the equation (3), when the electric field changes (sign), for the Hamiltonian to stay constant, the EDM (sign) should also change. Conversely, when the EDM changes, electric field should also change. In the observations discussed in the ”Transformations on a Neutron” subsection, it is observed that CP is violated according to the observations of time and parity transformations. The reason is that in the time transformation, electric field and spin (also EDM) behave oppositely: electric field stays the same but the spin; hence the EDM changes. In the parity transformation, EDM stays the same but the electric field changes. However, they should have both changed for the Hamiltonian to stay constant. On the other hand, the magnetic field and spin behave the same way in both transformations. Eventually, neutron EDM violates this but the magnetic dipole does not violate it. Therefore, the assumption is proven wrong. It can be concluded that neutron with an EDM cannot exist.

The θ parameter

The Lagrangian that describes how particles (i.e. gluons and quarks) interact through strong forces is given by the following equation with Einstein notation (μ, ν, α, β and a are repeated indices that refer to a summation):

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_f \bar{\psi}_f(i\gamma_\mu D^\mu - m_f)\psi_f + \frac{\theta g^2}{32\pi^2}\epsilon^{\mu\nu\alpha\beta}F^a_{\mu\nu}F^a_{\alpha\beta} \quad (4)$$

Even though this equation looks complicated to fathom, the crucial part to notice is that the θ term parametrizes CP violation in this equation, because it is proportional to the neutron EDM, which was proven to violate CP symmetry.

However, in the experiments, this angle is found to be very close to 0 ($\bar{\theta} \lesssim 10^{-10}$) without a certain reason [1]. The solution to the problem on why it should be zero is the ”Axion” solution, which was proposed by Roberto Peccei and Helen Quinn.

AXION SOLUTION

What is Axion?

Axion is the prime particle candidate for dark matter. The reason is that if the dark matter particle is a lowmass boson (i.e. force carrier), it may permeate the universe in a state which is cold, slow-moving and wavelike; similar to the axion’s field [1]. It has a pseudo-scalar nature, indicating that it changes sign under a parity transformation. It is under the field of quantum chromodynamics (QCD), which is the study of the strong force interactions between quarks and gluons. According to the standard model, quarks are particles that constitute protons and neutrons, and gluons are the force carriers that bind quarks together. Axion is considered as a solution to the CP Problem. The axion field is proportional to the θ term, making the angle dynamic rather than a constant. The angle takes values between 0 and 2π. Because of the ”spontaneous symmetry breaking” (which will be discussed in the following subsections) it undergoes, the θ term takes the value 0 every time and the neutron EDM vanishes. That is why axion is a solution to the CP Problem. In the equation (4), θ is taken as 0. Hence, the CP Problem is solved [1].

Some Symmetries

To introduce axion to the standard model, Roberto Peccei and Helen Quinn have proposed a symmetry: Pecce-Quinn (PQ) symmetry. This is a global axial U(1) symmetry, where the ”global” means symmetry transformation being the same everywhere and every time. ”Axial” means symmetry transformation acts differently on left-handed & right-handed particles, where the handedness describes the relative orientation of the particle's spin and velocity. Furthermore, U(1) symmetry transformation describes the symmetry where there is a rotation about a single axis. Axion is a field that obeys this symmetry [1].

Oscillatory Field

The axion field potential behaves periodically; its field is described by a cosine function as follows:

$$V(a(x)) \propto -\cos\left(\bar{\theta} + \frac{a(x)}{f_a}\right) \, (5)$$

where V (a) is the axion potential, a(x) is the axion field, $\bar{\theta}$ is the CP-violating QCD vacuum angle (it is with a bar because it includes additional violating angles resulting from the quark mass matrix in addition to θ), and f_a is the axion decay constant [1]. It is assumed that the shape of the axion potential is described by the dilute instanton cosine-like gas approximation [10]. Instantons are quantum tunneling effects that give axions mass when they're scarce and not interacting with each other [11].

SOMBRERO POTENTIAL

What is Sombrero Potential?

Sombrero potential is a 2 dimensional polar coordinate analog to the:

$$sinc\,(x) = \frac{\sin x}{x} (6)$$

function. This demonstrates its oscillatory property, which is related to the axion field. The reason why it is called ”Sombrero” is that it looks like a Mexican hat. This polar coordinate system valley has infinitely many equivalent minima points, which demonstrates its continuous symmetry. Sombrero potential is related to the “spontaneous symmetry breaking” of the axion field [1]. It is seen in the following figure (Fig. 4):

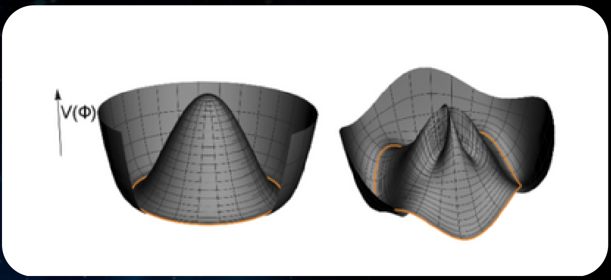


Fig 4. Sombrero Potential [1]

Spontaneous Symmetry Breaking

Spontaneous symmetry breaking can be illustrated using the parabola illustration: The vertex of a parabola lies on the axis of symmetry of the parabola. It is symmetrical with respect to the right and left sides of the axis of symmetry. If another point on the parabola is chosen other than its vertex, this point will not lie on the axis of symmetry. Therefore, the symmetry will be broken. Once the axion field picks one point on the orange ring in Fig. 4, it is called ”spontaneously broken symmetry”. Furthermore, the point is chosen to make θ=0 and choose the most stable point (i.e., where the energy is minimum). Since these minimum energy points are infinitely many, the field is continuously symmetric. When the field is pushed around the circle, the Nambu-Goldstone boson, which is an axion, emerges as a result of the broken symmetry. In the following section, scenarios concerning when and how axion dark matter occurred will be discussed [1].

EARLY UNIVERSE AND AXION

Introduction

In the early universe, the universe was flat, high in temperature, homogeneous, and isotropic, meaning that its properties were independent of the direction from which it was measured [12]. According to Friedmann, temperature was at its maximum at the beginning of the Big Bang. The scientists have proposed two scenarios for the origin of axion dark matter [1].

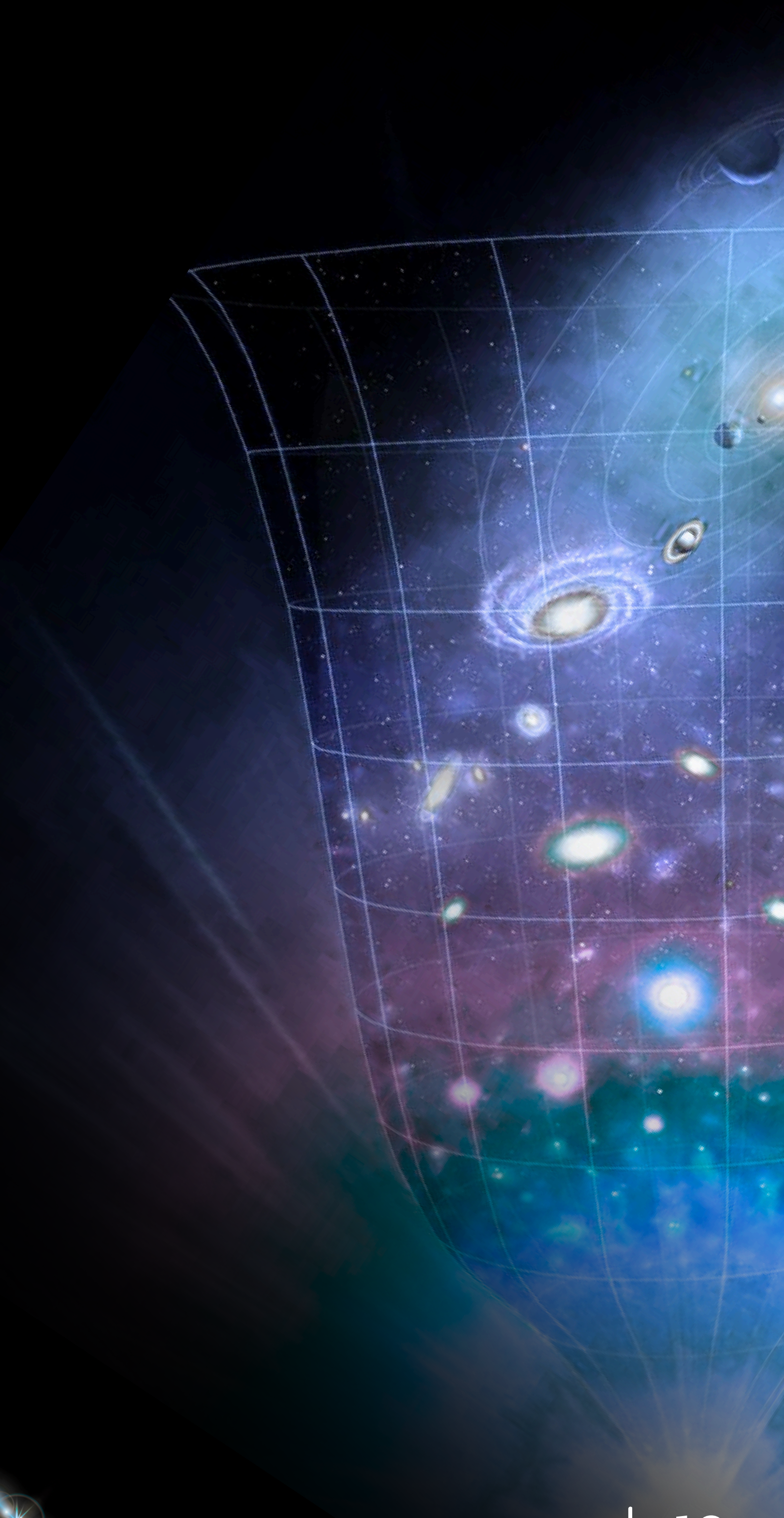
Scenario 1

One of them investigates the time before the Big Bang. In this scenario, the universe is homogeneous and smooth. Axion's evolution is only dependent on the wave equation of the universe expanding. As the universe keeps on expanding, the axion field oscillates. Over time, these oscillations dampen, hence the energy decreases. This damping turns into axion dark matter [1].

Scenario 2

The other scenario concerns the duration of the Big Bang. In this scenario, the axion field may not be the same everywhere. This is because of some cosmic strings, which are topological, thin, line-like defects. They emit gravitational waves and carry energy. Another reason is the existence of domain walls. They are sheet-like structures. They separate two regions of space where a field settles into different vacuum states.

Considering these phenomena, because of the axion field compositions that appear between regions where the axion field has dissimilar values, this axion field may decay into axion dark matter [1].



CONCLUSION

In this article, dark matter, the expansion of the universe, the CP Problem, and its axion solution have been examined. Axion is one of the most prominent theories as a candidate for dark matter. It is related to the fundamental symmetries in the universe. Its relation to cosmology and the early universe gives rise to two different scenarios: the axion's origin before or during the Big Bang. There are many experiments to understand dark matter and its relation to the axion. The microwave cavity haloscope is an example designed to detect axion dark matter by converting axions into photons when a strong magnetic field is present [1]. Even the axion's mass is not known very accurately because of the limitations of equipment, time, and resources. However, by interconnecting interdisciplinary approaches using engineering and other areas, it is believed that this research area will continue to improve.

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